MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2022/2023 (TERM ID 2220)

EEM1026 – ENGINEERING MATHEMATICS II

(RE/ME)

03 JULY 2023
9.00 a.m. – 11.00 a.m.
(2 Hours)

INSTRUCTIONS TO STUDENTS

1. This question paper consists of five pages including cover page and appendix with four questions only.

2. Attempt **ALL** questions. The distribution of the marks for each question is given.

3. Please write down all your answers in the answer Booklet provided. Show all relevant steps to obtain maximum marks.

Question 1

a) Suppose that the mean daily working hours of a population of workers is 9 hrs. The daily working hours of a sample of 19 workers produces a standard deviation of 1.8 hrs. What is the probability that the average working hours is at least 8? Assume the working hours to be approximately normal.

[5 marks]

b) In 2019, mean income in Malaysia is said to be 7901 Malaysian Ringgit (MYR). You would like to test whether the mean income is still the same. A sample of 250 randomly selected individuals provides a mean of 8050 MYR and standard deviation of 1250 MYR. Perform a test of hypothesis using test statistic at significance level of 0.05. Please draw the distribution curve and label the critical points as well as test statistic value.

[10 marks]

c) Suppose that the market for old cars comprise five different brands which is said to be equally distributed. You have observed the data as follows:

Car	Brand A	Brand B	Brand C	Brand D	Brand E
Frequency	35	43	54	48	20

Is the market for old cars equally distributed? Perform a χ^2 goodness-of-fit test. Use a significance level of 5%.

[10 marks]

Question 2

a) Solve the following linear differential equation using the method of undetermined coefficients

$$y'' + 3y' - 10y = e^{2x} + 10x$$

[13 marks]

b) Solve the differential equation $y'' + x^2y = 0$ in the form of a power series about the point $x_0 = 0$. Find up to coefficient c_5 of the series.

[Hint: $y = \sum_{n=0}^{\infty} c_n x^n$]

[12 marks]

Continued...

Question 3

a) By using Laplace transform methods, solve the following linear differential equation

$$y'' + 3y' + 2y = 12te^{-5t};\ y(0) = 0,\ y'(0) = 0$$

[17 marks]

b) By using the Fourier transform properties and Fourier transform pairs, find the Fourier transform of

i) $x(t) = te^{-2t}\sin(3t)\,u(t)$

ii) $x(t) = e^{-t}u(t) * u(t)$

iii) $x(t) = e^{-2|t-1|}$

[3+3+2 marks]

Question 4

Given the following initial-boundary value problem of a heat equation:

$$\text{PDE:} \quad u_t = k^2 u_{xx} \text{ for } 0 < x < L, t > 0$$

$$\text{BCs:} \ u_x(0,t) = 0, u_x(L,t) = 0 \text{ for } t > 0$$

$$\text{IC:} \quad u(x,0) = f(x) \text{ for } 0 < x < L$$

You are provided with the following information by using method of separation of variables:

$$u(x,t) = F(x)G(t)$$

$$\frac{G'}{k^2 G} = \frac{F''}{F} = \lambda$$

a) Show that the general solution is

$$u(x,t) = A_0 + \sum_{n=1}^{\infty} A_n \cos\left(\frac{n\pi}{L}x\right) e^{-\left(\frac{n\pi}{L}\right)^2 k^2 t}$$

[Hint: Consider only the cases for $\lambda = 0$ and $\lambda = -p^2$]

[17 marks]

b) From part a), suppose $f(x) = x$, show that the solution is

$$u(x,t) = \frac{L}{2} + \sum_{n=1}^{\infty} \frac{2L}{(n\pi)^2}((-1)^n - 1)\cos\left(\frac{n\pi}{L}x\right) e^{-\left(\frac{n\pi}{L}\right)^2 k^2 t}$$

[8 marks]

End of Paper

Appendix

Table 1: Laplace Transform Pairs

$f(t) = L^{-1}[F(s)]$	$F(s) = L[f(t)]$	$f(t) = L^{-1}[F(s)]$	$F(s) = L[f(t)]$
$\delta(t)$	1	$\sin(\omega t)$	$\frac{\omega}{s^2+\omega^2}$
$\delta(t-a)\,,\ a \geq 0$	e^{-as}	$\cos(\omega t)$	$\frac{s}{s^2+\omega^2}$
1 or $u(t)$	$\frac{1}{s}$	$\sinh(\omega t)$	$\frac{\omega}{s^2-\omega^2}$
$u(t-a)\,,\ a \geq 0$	$\frac{e^{-as}}{s}$	$\cosh(\omega t)$	$\frac{s}{s^2-\omega^2}$
$(t-a)u(t-a)\,,\ a \geq 0$	$\frac{e^{-as}}{s^2}$	$e^{-kt}\sin(\omega t)$	$\frac{\omega}{(s+k)^2+\omega^2}$
t^n	$\frac{n!}{s^{n+1}}\,,\ n = 1,2,3,\ldots$	$e^{-kt}\cos(\omega t)$	$\frac{s+k}{(s+k)^2+\omega^2}$
e^{at}	$\frac{1}{s-a}$	$e^{-kt}\sinh(\omega t)$	$\frac{\omega}{(s+k)^2-\omega^2}$
$t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}}\,,\ n = 1,2,3,\ldots$	$e^{-kt}\cosh(\omega t)$	$\frac{s+k}{(s+k)^2-\omega^2}$

Table 2: Laplace Transform Properties

Property	$f(t) = L^{-1}[F(s)]$	$F(s) = L[f(t)]$
Linearity	$af_1(t)+bf_2(t)$	$aF_1(s)+bF_2(s)$
Time shifting	$f(t-t_0)u(t-t_0), t_0 \geq 0$	$F(s)e^{-st_0}$
Frequency shifting	$e^{at}f(t)$	$F(s-a)$
Time differentiation	$f'(t)$	$sF(s)-f(0)$
	$f''(t)$	$s^2F(s)-sf(0)-f'(0)$
Time integration	$\int_0^t f(\tau)d\tau$	$\frac{1}{s}F(s)$
Frequency differentiation	$t^n f(t)$	$(-1)^n\frac{d^n}{ds^n}F(s)$
Frequency integration	$\frac{f(t)}{t}$	$\int_s^\infty F(\tilde{s})d\tilde{s}$
Time Convolution	$f_1(t) * f_2(t)$	$F_1(s)F_2(s)$

Table 3: Fourier Transform Pairs

	$x(t) = F^{-1}[X(w)]$	$X(w) = F[x(t)]$
1.	1	$\sqrt{2\pi}\delta(w)$
2.	$\delta(t)$	$1/\sqrt{2\pi}$
3.	e^{iw_0t}	$\sqrt{2\pi}\delta(w-w_0)$
4.	$e^{-at}u(t)$, $a>0$	$\frac{1}{\sqrt{2\pi}}\frac{1}{a+iw}$
5.	$e^{at}u(-t)$, $a>0$	$\frac{1}{\sqrt{2\pi}}\frac{1}{a-iw}$
6.	$e^{-a\|t\|}$, $a>0$	$\frac{1}{\sqrt{2\pi}}\frac{2a}{w^2+a^2}$
7.	$u(t)$	$\frac{1}{\sqrt{2\pi}}\left[\pi\delta(w)+\frac{1}{iw}\right]$
8.	$t^ne^{-at}u(t)$, $a>0$	$\frac{1}{\sqrt{2\pi}}\frac{n!}{(a+iw)^{n+1}}$
9.	$e^{-at}\sin(bt)u(t)$, $a>0$	$\frac{1}{\sqrt{2\pi}}\frac{b}{(a+iw)^2+b^2}$
10.	$e^{-at}\cos(bt)u(t)$, $a>0$	$\frac{1}{\sqrt{2\pi}}\frac{a+iw}{(a+iw)^2+b^2}$
11.	$e^{-at}\sinh(bt)u(t)$, $a>0$	$\frac{1}{\sqrt{2\pi}}\frac{b}{(a+iw)^2-b^2}$
12.	$e^{-at}\cosh(bt)u(t)$, $a>0$	$\frac{1}{\sqrt{2\pi}}\frac{a+iw}{(a+iw)^2-b^2}$
13.	e^{-t^2}	$e^{-w^2/4}/\sqrt{2}$

Table 4: Fourier Transform Properties

Property	**Operation**
Linearity	$af(t)+bg(t) \Leftrightarrow aF(w)+bG(w)$
Scaling ($a \neq 0$)	$x(at) \Leftrightarrow \frac{1}{\|a\|}X\left(\frac{w}{a}\right)$
Time Shifting	$x(t-t_0) \Leftrightarrow e^{-iwt_0}X(w)$
Frequency Shifting	$e^{iw_0t}x(t) \Leftrightarrow X(w-w_0)$
Time Convolution	$f(t)*g(t) \Leftrightarrow \sqrt{2\pi}G(w)F(w)$
Time Differentiation	$\frac{d^nx}{dt^n} \Leftrightarrow (iw)^nX(w)$
Frequency Differentiation	$t^nx(t) \Leftrightarrow i^n\frac{d^nX}{dw^n}$